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**225. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.**

Determine the sides of a triangle, given the lengths of (1) the three altitudes, (2) the three medians, (3) the radii of the escribed circles, (4) the radius $R$ of the circumscribed circle and any two of the three quantities $r$=radius of inscribed circle, $s$=semi-perimeter, $\triangle$ =area.

### Solution by the PROPOSER.

(1) Let $A$ be the altitude on side $a$, $B$ on side $b$, $C$ on side $c$.

$\therefore \triangle=\sqrt{[s(s-a)(s-b)(s-c)]}=\frac{1}{2}aA=\frac{1}{2}bB=\frac{1}{2}cC$,

$b=a(A/B)$, $c=a(A/C)$, $2s=a+b+c=a(1+A/B+A/C)$,

$2(s-a)=a(-1+A/B+A/C)$, $2(s-b)=a(1-A/B+A/C)$,

$2(s-c)=a(1+A/B-A/C)$.

$\therefore a=2/AW, b=2/BW, c=2/CW$, $W=\sqrt{[(1/A+1/B+1/C)(-1/A+1/B+1/C)(1/A-1/B+1/C)(1/A+1/B-1/C)]}$.

(2) Let $A$ be the median to side $a$, etc.; $O$ the intersection of the medians. From the triangle with base $a$ and vertex $O$, we have

$$(\tfrac{2}{3}C)^2+(\tfrac{2}{3}B)^2=2(\tfrac{1}{3}A)^2+2(\tfrac{1}{2}a)^2, \text{ whence } a^2=\tfrac{8}{9}B^2+\tfrac{8}{9}C^2-\tfrac{4}{9}A^2.$$

Similarly, $b^2=\frac{8}{9}A^2+\frac{8}{9}C^2-\frac{4}{9}B^2$, $c^2=\frac{8}{9}A^2+\frac{8}{9}B^2-\frac{4}{9}C^2$.

(3) With the usual notation for the radii of the escribed circles,

$$r_1=\frac{\triangle}{s-a},\ r_2=\frac{\triangle}{s-b},\ r_3=\frac{\triangle}{s-c},\ \triangle^2=rr_1r_2r_3,\ \triangle=rs.$$

$\therefore b+c-a=2s-2a=2\triangle/r_1$, $a-b+c=2\triangle/r_2$, $a+b-c=2\triangle/r_3$.

$\therefore a=\triangle(1/r_2+1/r_3)$, $b=\triangle(1/r_1+1/r_3)$, $c=\triangle(1/r_1+1/r_2)$.

$\therefore s=\triangle(1/r_1+1/r_2+1/r_3)$, $1/r=1/r_1+1/r_2+1/r_3)$,

$$\triangle^2=rr_1r_2r_3=\frac{r_1r_2r_3}{(1/r_1+1/r_2+1/r_3)},\ \triangle=\frac{r_1r_2r_3}{\sqrt{(r_1r_2+r_1r_3+r_2r_3)}}.$$

$$\therefore a=\frac{r_1(r_2+r_3)}{\sqrt{(r_1r_2+r_1r_3+r_2r_3)}},\ b=\frac{r_2(r_1+r_3)}{\sqrt{(r_1r_2+r_1r_3+r_2r_3)}},\ c=\frac{r_3(r_1+r_2)}{\sqrt{(r_1r_2+r_1r_3+r_2r_3)}}$$

(4) The three cases are equivalent since $\triangle=rs$. Let then $R$, $r$, $s$ be given. The problem may be reduced to (3) since $r_1$, $r_2$, $r_3$ are the roots of

$$x^3-(r+4R)x^2+s^2x-rs^2=0.$$

To determine $a$, $b$, $c$ directly, we employ $a+b+c=2s$, $abc=4\triangle R=4rsR$, $(s-a)(s-b)(s-c)=\triangle^2/s=r^2s$.

$\therefore s^3-s^2(a+b+c)+s(ab+ac+bc)-abc=r^2s$, $ab+ac+bc=s^2+r^2+4rR$. Hence $a$, $b$, $c$ are the roots of $y^3-2sy^2+(s^2+r^2+4rR)y-4rsR=0$.

**Also solved by J. E. Sanders, J. Scheffer, and G. B. M. Zerr.**